# Black-Scholes Model: Documentation and Mathematical Formulas

## Introduction

The Black-Scholes model is a mathematical model for pricing an options contract. The model was developed by Fischer Black, Myron Scholes, and Robert Merton in 1973. It is widely used for the valuation of European-style options.

## Assumptions of the Black-Scholes Model:

1. The stock price follows a geometric Brownian motion with constant drift and volatility.  
2. The option is European and can only be exercised at expiration.  
3. No dividends are paid out during the life of the option.  
4. Markets are efficient, meaning that there are no arbitrage opportunities.  
5. No transaction costs or taxes.  
6. Risk-free interest rate is constant and known.

## Black-Scholes Formula for European Call and Put Options

### Variables:

- S: Current stock price (underlying price)  
- K: Strike price of the option  
- T: Time to maturity (in years)  
- r: Risk-free interest rate (annual)  
- σ: Volatility of the stock (annual standard deviation of log returns)  
- d1 and d2: Intermediate calculations  
- N(x): Cumulative distribution function of the standard normal distribution

### Formulas:

1. Intermediate calculations:  
 d1 = (ln(S / K) + (r + 0.5 \* σ²) T) / (σ √T)  
 d2 = d1 - σ √T  
  
2. European Call Option Price:  
 C = S N(d1) - K e^{-rT} N(d2)  
  
3. European Put Option Price:  
 P = K e^{-rT} N(-d2) - S N(-d1)

## Interpretation of Variables:

- N(d1): Probability that the option will be exercised, adjusted for the time value of money.  
- S N(d1): Present value of receiving the stock if the option is exercised.  
- K e^{-rT} N(d2): Present value of paying the strike price if the option is exercised.

## Delta of an Option:

The delta (Δ) of an option measures the sensitivity of the option's price to a small change in the price of the underlying asset. It is the first derivative of the option price with respect to the underlying price.

### Delta formulas:

- Delta for Call Option:  
 Δ\_call = N(d1)  
  
- Delta for Put Option:  
 Δ\_put = N(d1) - 1

## Python Implementation:

Here is a Python function to calculate the Black-Scholes price of a European call or put option:

```python  
import numpy as np  
from scipy.stats import norm  
  
def black\_scholes\_price(S, K, T, r, sigma, option\_type='call'):  
 d1 = (np.log(S / K) + (r + 0.5 \* sigma \*\* 2) \* T) / (sigma \* np.sqrt(T))  
 d2 = d1 - sigma \* np.sqrt(T)  
  
 if option\_type == 'call':  
 price = S \* norm.cdf(d1) - K \* np.exp(-r \* T) \* norm.cdf(d2)  
 else:  
 price = K \* np.exp(-r \* T) \* norm.cdf(-d2) - S \* norm.cdf(-d1)  
  
 return price  
  
# Example usage:  
S = 100 # Current stock price  
K = 100 # Strike price  
T = 1 # Time to maturity (1 year)  
r = 0.05 # Risk-free interest rate (5%)  
sigma = 0.2 # Volatility (20%)  
  
call\_price = black\_scholes\_price(S, K, T, r, sigma, option\_type='call')  
put\_price = black\_scholes\_price(S, K, T, r, sigma, option\_type='put')  
  
print(f"Call Option Price: {call\_price}")  
print(f"Put Option Price: {put\_price}")  
```

## Summary:

The Black-Scholes model provides a theoretical estimate of the price of European call and put options. It requires inputs such as the current stock price, strike price, time to maturity, risk-free interest rate, and volatility. The model helps in understanding the factors that influence option prices and is widely used in financial markets for pricing and hedging purposes.